Algorithmic Design. Structuralism Reloaded?

A recent special issue of the journal Arch+, titled "Entwurfsmuster", design pattern, examines the changes to the design process caused by the ongoing digitalization of the discipline of architecture. In the issue, the editors, Nikolaus Kuhnert and Anh-Linh Ngo, argue that the digital approach should be seen primarily as a disclosure of constantly reappearing patterns of thinking that govern the design process. Therefore, digitalization offers, above all, the chance to clarify the methodology of design in architecture, due to the necessary formalization of relationships inherent in digital design.

Such thinking in relationships bears some resemblance to the concept of structuralism in architecture that is often associated with a critical view of the CIAM functionalism of the 1950s poststructural architecture movement arising from Team 10 and the related architectural phenomena of the 1960s and 1970s. In both approaches toward architectural design, the notion of structure is understood to be an organizational pattern constituted solely by the immanent network of spatial connections among the component elements. In general, digital design has reanimated the theoretical interest in questions of structure in recent years, because "with the digitalization of architecture, which is relationally and parametrically comprehensible, the technical prerequisites for a structural reading of architecture exist. This time, however, they are founded no longer in terms of linguistic analysis, but rather geometry."

This difference in the conceptualization of structural thinking, however, poses the question of comparability between a contemporary design methodology, referred to as “neostructuralism with a digital imprint” by Kuhnert and Ngo, and a historically defined movement in architecture. This is especially the case because an agreement exists neither on structural methodology nor on the notion of structure itself. Rather, different understandings of the structural concept are found in various investigations, adapted in each case to the relevant context – hence, a “plurality of forms of structuralism” exists, whose sole commonality is the general objective: “to identify and understand underlying structures within a given field of interest.” In general, structuralist works can be understood as an effort to reveal the underlying rationales associated with a figure, whereby the conception of such a figure generally requires specialized knowledge and the structural question always has a specialized character.

The following study, therefore, focuses on building up a common understanding of the notion of structure in order to make neostructuralism comparable with a digital imprint and the structural architectural movement. The comparison will be based on a perspective on architectural design as a conceivable expression of chains of decisions that control the underlying design process. Central here, however, is not the semantic interpretation of this chain of decisions in a cultural framework – as is otherwise common in architecture – but rather a syntactical observation of these chains. Such a process of observation is motivated by developments in the humanities, in

1. Fundamental structures in mathematics.
particular the work of Claude Lévi-Strauss on anthropological structuralism, including his work *Mythologiques*, an investigation of the cultural evolution of myths. The focus of his study is the link between the elements of mythological tales – not an examination of the content of these stories, however. In his view, the meaning of myth can be found not in its isolated components, but in the manner in which they are combined, making up bundles of relations called by Lévi-Strauss "gross constituent units" or *mythèmes*. It is this reduction that enables him to reveal the intellectual structures of the story and makes it possible to compare myths from different cultural circles. Similarly, the design process is to be understood as an architectural myth to be supplemented with a structuralist observation.

*ta mathēmata*

Lévi-Strauss noted that a specification of the concept of structure represents a general questioning of the methodology in the sciences and is not specifically restricted to structuralism. For this reason, it is no surprise that all attempts to define the concept possess a general character. For example, the anthropologist Alfred R. Radcliffe-Brown describes the idea of structure as follows: "When we use the term structure we are referring to some sort of ordered arrangement of parts or components. A musical composition has a structure, and so does a sentence. A building has a structure, so does a molecule or an animal. The components or units of social structure are persons, and a person is a human being considered not as an organism but as occupying position in a social structure."

In this way, a structure can be understood as a pattern that results from the network of formal relationships between elements of a set of examined objects. A contemporary mathematical understanding is reflected in the concept of structure, because "a structure is any set of objects (also called elements) along with certain relations among those objects". This contextual proximity to mathematics is not the result of chance; mathematics is the science of patterns, whereby the fundamental patterns result from the formalization of human perception.

The world is a construct whose main reference point is the here and now of the body. It is from the body that surrounding reality is perceived, structured, and accessed. Mathematics takes part in this act of orientation in the world through an active and constructive act of appropriating the environment by setting boundaries by means of structured differentiation. According to Heidegger, this human capability of recognizing order is the "truly mathematical", which stems from the original meaning of the Greek *ta mathēmata*. "The mathēmata are the things insofar as we take cognizance of them as what we already know them to be in advance. ... Therefore we do not first get it out of things, but, in a certain way, we bring it already with us."

According to Heidegger, the concept of numbers does not arise from experience of counting elements in a set, but rather the concept of numbers must already exist in order to determine the number of elements. Likewise, the process of structuring is not such an abstract, logical construct of order, but rather describes human's internal capacity to perceive sets.

Mathematical Structures

These fundamental perspectives for sets are demonstrated in the more formalized manner of modern mathematics as well. Under the pseudonym Nicolas Bourbaki, a group of mainly French mathematicians worked on the systematic organization of mathematics since 1934. Bourbaki, building on the concept of set, attempted to generalize mathematical ideas that were already recognized, and thus systematize in such a way that the inherent dependencies between different branches of mathematics would be clear. It would be clear if the same structures were revealed everywhere. The Bourbaki project, which ran over several decades, made evident that three types of organizational patterns form the fundamental and reoccurring structures in mathematics: organizational structures, algebraic structures, and topological structures.

All types of structure deal with a specific examination of the elements of a given set. For organizational structures, the links between the subsets are emphasized; that is, it is a question of the subdivision, of hierarchy and quantification. For algebraic structures, the elements themselves and the network of relationships within a set are the focus; that is to say, the linking of elements, their combinability. For topological structures, the neighboring links of elements or subsets are the
focus; that is, the concern is questions of similarity, proximity, qualitative comparability. To put it simply, organizational structures are about different ways of counting the elements of a set, algebraic structures are about doing calculation with these elements, and topological structures are about the geometric organization of the elements.

The three types of mathematical structures thus formalize elemental perspectives of sets. These are linguistic specifications of patterns of human perception: *ta mathēmata*. As patterns of perception, these structures are not bound to the mathematical context. Rather, mathematics as a scientific discipline only forms a specific context within which these figures of thought become visible. Organizational structures, algebraic structures, and topological structures can be understood as exemplary descriptions of patterns of human involvement with the environment, as typological representations of a modus of thinking in structures internal to humans.

**Anthropological Structuralism**

In the work of Lévi-Strauss, the movement from anthropological structuralism to mathematics is not surprising. In 1942, he turned to the mathematician Jacques Hadamard to request support for the formulation and specification of rules according to which various tribes married their partners. The analysis of systems of relationships has been central to the understanding of cultural groups since the first anthropological studies in the 19th century. Relationships in this case include formal aspects such as marriage customs and the transfer of property as well as the structure of family relationships. Lévi-Strauss maintained hope that the mathematization of relationship connections would lend the social sciences strict scientific potential somewhat like physics.

Hadamard, however, had no interest in collaboration. One year later, Lévi-Strauss met André Weil, one of the co-founders of the Bourbaki group. Weil solved the problem of marriage using an algebraic structure; he ignored the marriage itself and concentrated instead on the changes in relationships that resulted from the marriage. He noted the potential marriages using so-called permutation groups, or combined exchanges between elements, in which men and women of every tribe were assigned numbers. After Weil recorded the tribes as sets and their link was expressed in permutations, he was able to prepare the complex marriage rules on the basis of the group structure of the tribes. This presented Lévi-Strauss with the chance to add a mathematical appendix from André Weil to the first part of his book *Les structures élémentaires de la parenté*. In this way, Lévi-Strauss was able to demonstrate the close connection between anthropological and mathematical structures, which he had been striving for. In later studies, such as those that addressed the analysis of myths, Lévi-Strauss always tried to apply mathematical structures; as scientific philosopher Michel Serres noted in 1977: "The most productive methods related to mythical text in general are arranged algebra and, to be more precise, combinatorial algebra."

The relatively direct applicability of elementary mathematical structures to questions of ethnology and sociology is based primarily on the fundamental assumptions of anthropological structuralism. This, in turn, is based on the principle that human needs are the same everywhere; as a result, a differentiated observation of the subject is unnecessary for the study of a societal group. The subjects of a group can in this way be understood as a collection of equal elements in a set. According to the structuralist understanding, the social life stems not from the sum of the individuals, but from the regularity of the network of relationships between subjects in a community as a decentralization of the subject. The elements of a set thus interact according to specific regularity. It is this regularity of specific networks of relationships that Weil described using algebraic structures.
Structuralism in Architecture

The structuralist assumptions of the atemporal equality of humankind lend the described social structures an archetypal character. In the criticism of developments pervaded by functionalism and rationalism in postwar Europe, anthropological structuralism presented the opportunity for thinking back to true human needs. With the goal of a human-oriented architecture, this led to a repeated examination of the architecture of various archaic cultures in the context of the CIAM and later in the publication *Forum*; the primitive architecture, which was understood in a certain manner, became a symbol for a way of life reflected therein. This way of life comes to us through all the centuries and is deeply rooted in human and cosmic nature. ... Today, this creates – with urgent necessity – the possibility to push modern architecture to even higher levels.

For such a heightening of modern architecture, elevated by anthropological structuralism, the issue is not rebuilding archaic styles of living, but rather creating the contemporary form of the regular network of relationships, that is, in order to interpret the temporally and spatially adapted individual expression of the archetypal structure. Despite the differences in materialization and design vocabulary, there are significant similarities in the spatial organization of the traditional tribal settlement of the Musgum in northern Cameroon and the Halen development by Atelier 5, built in 1961. Both are buildings oriented inward in an isolated environment with a controlled entrance situation. Both buildings consist of similar living units, which are grouped around a central community space. The living units differ with respect to clearly placed elements in space, in one case by the cooking location, in another by separation walls.

According to Hertzberger, for structuralism in architecture, along with the decentralization of the subject and the archetypical structures, the question of the transformation of these structures...
and the associated possibility of an architectural adaptation is of central importance. 26 “Every solution in any location and at various times is an interpretation of the archetypical, in general and in particular, similar to the individual application of a formula.” 27 In the design process, the archetypal structure is understood as an eidetic form of the network of relationships. The primary spatial form of this set of relationships, its gestalt, occurs through the purposeful insertion of organizing elements. It is these primary elements that generate the spatial framework for the cooperation of the users.

The interaction of archetypal structure and primary elements becomes a conceptual diagram of architecture. This diagram enables a multitude of possible forms as a constructed reality. In the Diagoon houses by Herman Hertzberger, this openness of form becomes a central aspect of the architecture. The primary elements of the houses are described through three levels, which form a central, vertical space. Around this zone of visual and verbal exchange, the freely shapeable ground plan of the residential buildings is organized. The primary elements determine the fundamental organization of the houses, not only in the sense of support structures, but also and above all in a sociological sense; in doing so, they define the zones of consolidation, interaction, and communication.

Hertzberger’s design is guided by the idea of the production of the network of relationships and not by a concrete design vocabulary. The Diagoon houses display a topological design process of the plastic form of the archetypal structure of a family. This is made all the more clear in the design of the orphanage in Amsterdam by Aldo van Eyck. The primary elements form an axis of open interactions, the simultaneity of activity that is supported by the differing degrees of concentration. In stepping out of these zones, one enters a more private space, whose individuality is emphasized by the spatial scales. Here as well the realized form plays a subordinate role in comparison to the organization of the individual spaces. In this way, the architecture becomes an amorphous background to the social interaction. 28 A vagueuness in design is also demonstrated in the topological openness of the diagrams and the interpretation of the archetypical as emphasized by Hertzberger.

**Digital Architecture**

Such vagueuness in design and openness of thought, typical for structuralism, is foreign to the so-called digital architecture, which is architecture designed in large part by the use of the computer as main design tool. Computers are actively shaping the way we as users approach design questions. In general, human understanding of the world depends to a great degree on the interaction of the body with its environment. Every tool mediates this interaction because of its specific usage, thereby influencing the perception of the user and his way of thinking. 29 Computer-based design, therefore, leads to a specific form of reflection on architectural problems and this specific form is determined by the characteristics of the machine.

The versatility of a computer is built on the generality of its main constituents: a machine, hardware, manipulating data according to a set of instructions, software. The set of instructions defines a causal relationship between input in and output out and that, is why it can be viewed as mathematical function \( p \) with \( p(\text{in}) = \text{out} \). It is this abstract setting for a computing device that facilitates the principal question of computability; that is, for which function \( f \) does there exist a program \( p \) such that \( f(\text{in}) = p(\text{in}) \) for every valid input \( \text{in} \). In 1936, it was the mathematician Alan Turing who gave an abstract mathematical machine-like model of what it means for a function \( p \) to be computable as well as the description of what is now called a Turing machine. 30 This simple ab-
strict device is one of the earliest and most intuitive ways to make the basic idea of computability precise, and the underlying logic is closely connected to the later development of computers.31

Working with the computer brings about the necessity to think in causal links between input data and output data to formulate design actions as causal relationships between quantifiable entities. This necessity is often not obvious during the daily use of software. For example, to draw a straight line in a CAD environment appears to be a natural act: by activating an icon and selecting two points, the user forms a line. The conventional, graphic input format and the use of intuitive icons aids human-machine communication. At the same time, however, the act of forming a line follows the logic of mathematical description as a matter of principle; it discretely disassembles the act of drawing a line into a clearly defined sequence of events with the causal relationship \( p \) (defined by the icon) and the quantified input data \( in \) (the selected points in space) and results in the production of a line (the determined output \( out \)). This formal organization of a mathematical function into its constituent parts \(- p, in, and out \)- and its determinant character for the design process is more obvious in contemporary digital design methods like the parametrical work in an associative design environment such as Bentley’s Generative Component or the Grasshopper plug-in for Rhinoceros or in the algorithmically working defined design processes in a scripted environment.

The use of the computer as a design tool thus pervades architectural thought; it forces the quantification and causality linking of concepts and spatial ideas – that is, the precise specification of design ideas. The focus, then, is on formal strategies for the organization of data rather than the indeterminate and associative linking of conceptual images. That is why digital design is characterized by combinatorial and numerical questions and subsequently by a movement toward clearly conceivable algebraic and organizational structures for the purposeful manipulation of data. In contrast, quantitative thinking, which dominates the topological methods of examination in structuralism, is of less importance because such an approach toward design often cannot be quantified without loss of spatial implication. This does not mean that the design vocabulary in a digital design itself cannot be topological – that is, flexible, malleable, and easily deformable. This softness of form, however, is not generated by a vagueness of design method, but is the result of a quantified logic behind the soft form and the numerical control of the deformation process. The mere use of flexible geometry such as Non-Uniform Rational B-Splines (NURBS) in a digital design process does not signify topological thinking but rather the opposite: the exponential growth of deterministic mathematical relationships that govern the design and, therefore, a steering toward quantification.

Conclusion

There is a need for differentiation between architectural forms as such and the way of thinking that results in these kinds of architectural forms. This is true particularly for the concept of structuralism in architecture, which cannot be bound to a specific design vocabulary but rather is dependent on a specific form of thought. In general, forms of thinking can be seen as patterns of information processing. That is why mathematics as the science of patterns offers a unifying approach toward a characterization of different types of thinking. Mathematical concepts enable the
study of the relationship between different structuralisms that coexist in the discipline of architecture; they provide the "technical prerequisites for a structural reading of architecture" that is independent from the recent digitalization of architecture and the associated geometric reading of structures that Kühnert and Ngo were anticipating and that has been mentioned in the introduction.

Such a reading uncovers a strong topological line of thinking in Dutch structuralism. The associated openness and vagueness that are related with any form of topological thinking are visible in the design processes of architects such as Hertzberger or van Eyck. The underlying design process leaves its traces in an architecture that, despite its physical presence, remains vague; the "configurative design" challenged the users to play for their right to identity precisely by offering an architecture devoid of functional, iconographic or typological pedigree, an architecture antithetical to the idea of architecture par excellence.\(^33\)

Topological thinking is a form of thinking that is focused on qualitative relationships.\(^34\) The necessary blurriness associated with such an approach is not possible in a digital design process. The digital milieu necessitates precision in the formulation of internal logics of form and, therefore, is governed by a form of thinking focused on quantitative relationships. This turns architectural thinking into a survey of concepts and ideas that have to be quantified and transformed into a causal flow of information in order to define logically coherent design processes.

The difference in principle between qualitative thinking in a topological approach and quantitative thinking in a digital approach does not mean that the resulting architecture has to be different per se; differing lines of argumentation can lead to the same conclusion. However, quantitative and qualitative thinking mark the extreme positions of a spectrum of structural methodologies and corresponding understanding of the form-generating process. A qualitative design approach is characterized by a tendency from the general to the more specific, with the final form a result of a search process guided by constant reduction of vagueness. In contrast, a quantitative design approach is characterized by a tendency from the specific to the more general, with the final form a result of a controlled process of constant enhancement of primary shapes.

With respect to an architectural reading of social structures, these two types of thinking bear witness to very dissimilar perspectives on the human condition. The algebraic-topological mode of thinking in Dutch structuralism tries to accommodate the uncertainty of individual human behavior through qualitative vagueness, resulting in architectural space that appears as a neutral and uniform vessel in an amorphous global form. This form expresses the equality of humans through repetitive architectural elements such as the structural grid or space modules, which enable as background, as built diagram, the free unfolding of social interactions among the users. The algebraic-organizational mode of thinking of digital architecture, on the other hand, tries to avoid the uncertainty of individual human behavior through statistical quantification and rule-based modeling of behavioral patterns. This results in a performative emergence of architectural space with form incorporated into a fabric of geometrical links that determine the spatial disposition of the social structure. In other words, digital architecture is driven by a compulsion to frame and guide the social interactions of the users.

From this point of view it is apparent that Dutch structuralism and the so-called neostructuralism with a digital imprint are types of architectural thinking coming out of different lines of develop-
opment. Digital architecture is not rooted in the historic understanding of a linguistically motivated structuralism in architecture but rather in cybernetics and its pattern-based and quantity-based approach toward architecture.  

The digital does not offer the possibility for a systematic structural reading of architecture as implied by Kuhnert and Ngo in their introduction to “Entwurfsmuster”(). This is due to the fact that algebraic-topological forms of thinking cannot be formalized entirely in an algebraic-organizational form of description and, therefore, fall out of the digital realm. Consequently, divergent manifestations of structuralism exist simultaneously in architecture and, as a result, a “plurality of forms of structuralism” – as generally found in the sciences – is present in the discipline of architecture. Architecture and the debate over structuralism thus demonstrate the variety of human concepts of and thought about the world, and this variety is reflected in the mathematics of structural thinking.


4 Ibid.


7 From a scientific point of view, this statement describes only the scientific self-conception, which, since Galileo Galilei, has been reflected in the use of precise mathematical systems of notation to describe natural phenomena. In the humanities, structuralism was also understood as a possibility for objective specification and thus an approximation of the scientific understanding of the natural sciences. See John Forge, "Reflections on Structuralism and Scientific Explanation", in: Synthese, no. 130, 2002, 109–121, or Wolfgang Balzer, Ursulas Moulines, and Joseph D. Sneed, An Architectonic for Science: The Structuralist Program (Dordrecht, D. Reidel, 1987).

8 Mythologies comprised four volumes and was published between 1964 and 1971 under the titles Le Cru et le cuit (1964), Du miel aux cendres (1966), L'Origine des manières de table (1968), and L'Homme nu (1971).


16 Here the term "modern" points to the foundational crisis of mathematics in the early 20th century and the different attempts to find a proper logical foundation without inherent paradoxes or inconsistencies. This crisis has not been resolved, but the urgency of the question has faded away and has been replaced by a more pragmatic approach toward doing mathematics.

17 Between 1939 and 1998, the results of the group were published in the series Éléments de mathématique. This series of publications was extremely influential. Much of the mathematical terminology in use today as well as the concepts and methods of modern mathematics stem from the work of Bourbaki.

18 The successive treatment of these three types of mathematical structures was covered in the first three titles of the series Éléments de mathématique, which were published by the Bourbaki group under the pseudonym from 1939 to 1942.

19 As argued before, ta mathêmatikai results from the human cognitive apparatus. Various researchers, such as Lakoff and Núñez, therefore, suggest that mathematics has to be understood in cognitive terms. Inspired
by theoretical principles of embodied cognition and using mainly techniques from cognitive linguistics, they have proposed that idealized abstract technical entities in mathematics are created by the human imaginative mind via a very specific use of everyday body-grounded cognitive mechanisms. George Lakoff and Rafael Núñez, *Where Mathematics Comes From* (New York, Basic Books, 2000).


24 The members of Team 10 were among the driving forces behind the magazine *Forum*, and it was especially Aldo van Eyck who gave Dutch structuralism its anthropological orientation through several research trips to Africa and his general interest in humanist questions.


26 According to Herman Hertzberger, the essence of structuralism is demonstrated in the field of tension of archetypical structure and transformational freedom: "A characteristic of structuralism is the dialectics of change and remaining itself." Presentation at the symposium *Structuralism in Architecture & Urbanism Reloaded*, Munich, Nov. 19 to Nov. 21, 2009.


28 "One of Aldo van Eyck's programmatic formulae was that of a 'casteh organisé'. ... The ordering structure, mostly constructive grids, which might appear simplistic, can actually be seen as signifying the absence of an order and an invitation to the user to develop a more complex and socially rooted organization than architecture as such would be able to offer in the first place." Karin Jaschke, "Architecture as Artifice", in: *The Journal of Architecture*, no. 6, 2001, 135-144.


32 See footnote 1.


34 Lévi-Strauss's discourse on mythology reveals a clear movement toward a topological method of argumentation, a qualitative form of examination, through which the myths of different cultures can be compared in terms of their design and organization. Such a shift away from algebraic methods was necessary because, as Michel Serres has pointed out, "I have come to conclude that, like in the book from Claude Lévi-Strauss, we have exhausted all the potential of algebraic combinatorics; we can now exploit the potential of a different set of tools. I am not talking about a sister science. I am not identifying algebra as parallel to topology - that would be ridiculous, of course. I am only saying that we can take advantage of a set of tools from a related science." *Topologie: Zur Raumbeschreibung in den Kultur und Medienwissenschaften*, ed. Stephan Günzel (Gießen, transcript, 2007), 226.

35 For a detailed exploration of the role of cybernetics in architecture, see, for example, Antoine Picon, *Digital Culture in Architecture* (Basel, Birkhäuser, 2010).